Transformation of an equation by diminishing (or increasing) its roots by a constant:

Transformation of equation:

If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of the equation $f(x)=0$ then forming the equation whose roots are $\phi(\alpha_1), \phi(\alpha_2) \ldots \phi(\alpha_n)=0$ is called Transformation of the equation

Diminishing the roots of an equation:

If x is a root of the given equation and y, the root diminished by h then

$$x=y+h$$

Thus replacing x by $y+h$ in the given equation, we get the equation in y with the diminished roots.

① Diminish by 2 the roots of the equation $x^4+x^3-3x^2+2x-4=0$

2	1	1	-3	2	-4
	0	2	6	6	16
	1	3	3	8	$\boxed{12}$
	0	2	10	26	
	1	5	13	$\boxed{34}$	
	0	2	14		
	1	7	$\boxed{27}$		
	0	2			
	1	$\boxed{9}$			

$\therefore$ The required equation is

$$y^4+9y^3+27y^2+34y+12=0$$

(2) Find the equation whose roots are less than those $2x^3-7x^2+3x-5=0$ by 2

2	2	-7	3	-5
	0	4	-6	-6
	2	-3	-3	$\boxed{-11}$
	0	4	2	
	2	1	$\boxed{-1}$	
	0	4		
	2	$\boxed{5}$		

The new equation is $2y^3+5y^2-y-11=0$

(3) Find the equation whose roots are greater by unity than the roots of $x^3-5x^2+6x-3=0$

-1	1	-5	6	-3
	0	-1	6	-12
	1	-6	12	$\boxed{-15}$
	0	-1	7	
	1	-7	$\boxed{19}$	
	0	-1		
	1	$\boxed{-8}$		

$\therefore$ The new equation is $y^3-8y^2+19y-15=0$

4) Increase the roots of the equation $x^4+12x^3+56x^2+120x+91=0$ by 3 and hence solve the equation.

-3	1	12	56	120	91
	0	-3	-27	-87	-99
	1	9	29	33	$\boxed{-8}$
	0	-3	-18	-33	
	1	6	11	$\boxed{0}$	
	0	-3	-9		
	1	3	$\boxed{2}$		
	0	-3			
	1	$\boxed{0}$			

$\therefore$ The new equation is $y^4+0y^3+2y^2+0y-8=0$

$$y^4+2y^2-8=0 \qquad \frac{-8}{4 \mid -2}$$

$$(y^2+4)(y^2-2)=0$$

$y^2+4=0 \qquad y^2-2=0$

$$y^2=-4$$

$$y^2=\sqrt{-4}$$

$$y^2=\pm\sqrt{4\times -1}$$

$$y=\pm 2i$$

$$y^2-2=0$$

$$y^2=2$$

$$y=\pm\sqrt{2}$$

$\therefore$ Increased roots: $2i, -2i, \sqrt{2}, -\sqrt{2}$

Required roots are $-3+2i, -3-2i, -3+\sqrt{2}, -3-\sqrt{2}$

5 Find the equation whose roots are the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ increased by 2 and hence solve the

Solution.

-2	1	-1	-10	4	24
	0	-2	6	8	-24
	1	-3	-4	12	$\boxed{0}$
	0	-2	10	-12	
	1	-5	6	$\boxed{0}$	
	0	-2	14		
	1	-7	$\boxed{20}$		
	0	-2			
	1	$\boxed{-9}$			

The new equation is $y^4 - 9y^3 + 20y^2 = 0$

$$y^2(y^2 - 9y + 20) = 0$$

$y^2 = 0$

$\boxed{y = 0, 0}$

$y^2 - 9y + 20 = 0$ $\quad$ 20: $-4 \mid -5$

$(y-4)(y-5) = 0$

$y - 4 = 0$ $\quad$ $\boxed{y = 4}$

$y - 5 = 0$ $\quad$ $\boxed{y = 5}$

Increased roots are $0, 0, 4, 5$

The required roots are $0-2, 0-2, 4-2, 5-2$

$-2, -2, 2, 3$

(6) Diminish the roots of the equation $x^4-4x^3-7x^2+22x+24=0$ by 1 and hence solve the equation.

1					
	1	-4	-7	22	24
	0	1	-3	-10	12
	1	-3	-10	12	$\boxed{36}$
	0	1	-2	-12	
	1	-2	-12	$\boxed{0}$	
	0	1	-1		
	1	-1	$\boxed{-13}$		
	0	1			
	1	$\boxed{0}$			

The equation with the diminished roots is

$$y^4+0y^3-13y^2+0y+36=0$$

$$y^4-13y^2+36=0 \qquad \begin{array}{c} 36 \\ \hline -4 \mid -9 \end{array}$$

$$(y^2-4)(y^2-9)=0$$

$y^2-4=0$ $\qquad$ $y^2-9=0$

$y^2=4$ $\qquad$ $y^2=9$

$y=\pm 2$ $\qquad$ $y=\pm 3$

The roots of the equations are $2,-2,3,-3$

The roots of the original equation are

$\therefore$ The roots of the original equation

$2+1, \quad -2+1, 3+1, -3+1$

$3, -1, 4, -2$

Note:

Consider the equation is

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$$

For removing the second term $h = \frac{-a_1}{na_0}$

(1) Diminishing the roots by a suitable constant transform the equation $x^4 - 8x^3 + 19x^2 - 12x + 2 = 0$ into an equation in which the x^3-term is absent and hence solve the original equation.

Given $x^4 - 8x^3 + 19x^2 - 12 + 2 = 0$

$a_0 = 1, a_1 = 8, n = 4$

$$h = \frac{-a_1}{na_0} = \frac{-(-8)}{1 \times 4}$$

$$h = \frac{8}{4}$$

$$\boxed{h = 2}$$

2	1	-8	19	-12	2
	0	2	-12	14	4
	1	-6	7	2	6
	0	2	-8	-2	
	1	-4	-1	0	
	0	2	-4		
	1	-2	-5		
	0	2			
	1	0			

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The new equation is $y^4+0y^3-5y^2+0y+6=0$

$$y^4-5y^2+6=0 \qquad \begin{array}{c} 6 \\ -2 \mid -3 \end{array}$$

$$(y^2-2)(y^2-3)=0$$

$$y^2-2=0 \qquad y^2-3=0$$

$$y^2=2 \qquad y^2=3$$

$$y=\pm\sqrt{2} \qquad y=\pm\sqrt{3}$$

$\therefore$ The roots of the equations are $\sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}$

$\therefore$ The roots of the original equations are
$2+\sqrt{2}, 2-\sqrt{2}, 2+\sqrt{3}, 2-\sqrt{3}$

(8) Show that, on diminishing the roots of the equation $6x^4-43x^3+76x^2+25x-100=0$ by 2 it becomes a reciprocal equation and hence solve it.

2	6	-43	76	25	-100
	0	12	-62	28	106
	6	-31	14	53	$\boxed{6}$
	0	12	-38	-48	
	6	-19	-24	$\boxed{5}$	
	0	12	-14		
	6	-7	$\boxed{-38}$		
	0	12			
	6	$\boxed{5}$			

The new equation is $6y^4+5y^3-38y^2+5y+6=0$

$\div y^2$

$$6y^2+5y-38+\frac{5}{y}+\frac{6}{y^2}=0$$

$$6y^2+\frac{6}{y^2}+5y+\frac{5}{y}-38=0$$

$$6\left(y^2+\frac{1}{y^2}\right)+5\left(y+\frac{1}{y}\right)-38=0$$

$$y+\frac{1}{y}=t$$

$$y^2+\frac{1}{y^2}=t^2-2$$

$$6(t^2-2)+5t-38=0$$

$$6t^2-12+5t-38=0$$

$$6t^2+5t-50=0 \qquad \begin{array}{c|c} \multicolumn{2}{c}{-300} \\ \hline 20 & -15 \end{array}$$

$$6t^2-15t+20t-50=0$$

$$3t(2t-5)+10(2t-5)=0$$

$$(2t-5)(3t+10)=0$$

$$2t-5=0 \qquad 3t+10=0$$

$$2t=5 \qquad 3t=-10$$

$$\boxed{t=\frac{5}{2}} \qquad \boxed{t=\frac{-10}{3}}$$

1

2

case (i)

$$x+\frac{1}{x}=t$$

$$x+\frac{1}{x}=\frac{5}{2}$$

$$\frac{x^2+1}{x}=\frac{5}{2}$$

$$2(x^2+1)=5x$$

$$2x^2+2=5x$$

$$2x^2-5x+2=0 \qquad \begin{array}{c|c} \multicolumn{2}{c}{4} \\ \hline -1 & -4 \end{array}$$

$$2x^2-x-4x+2=0$$

$$x(2x-1)-2(2x-1)=0$$

$$(x-2)(2x-1)=0$$

$$x-2=0 \qquad 2x=1$$

$$\boxed{x=2} \qquad \boxed{x=\frac{1}{2}}$$

case (ii)

$$x+\frac{1}{x}=t$$

$$x+\frac{1}{x}=\frac{-10}{3}$$

$$\frac{x^2+1}{x}=\frac{-10}{3}$$

$$3(x^2+1)=-10x$$

$$3x^2+3=-10x$$

$$3x^2+10x+3=0 \qquad \begin{array}{c|c} \multicolumn{2}{c}{9} \\ \hline -1 & -9 \end{array}$$

$$3x^2+x+9x+3=0$$

$$x(3x+1)+3(3x+1)=0$$

$$(x+3)(3x+1)=0$$

$$x+3=0 \qquad 3x+1=0$$

$$\boxed{x=-3} \qquad 3x=-1$$

$$\boxed{x=\frac{-1}{3}}$$

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∴ The root of the equations are $2, \frac{1}{2}, 3, \frac{-1}{3}$

∴ The required roots are

$$2+2,\ \frac{1}{2}+2,\ -3+2,\ -\frac{1}{3}+2$$

$$4,\ \frac{5}{2},\ -1,\ \frac{5}{3}$$

1) Increase the roots of the equation $x^4+16x^3+83x^2+152x+84=0$ by 4 and hence solve it.

Roots: $-3, -6, -1, -7$

2) Diminish the roots of the equation $x^4-4x^3+7x^2-6x-10=0$ by unity and hence solve the equation

Roots: $1 \pm \sqrt{3},\ 1 \pm 2i$